Investigating Chance

Investigating Chance- This unit addresses probability, by examining students' everyday intuitions then providing opportunities for students to investigate, challenge, and refine their intuitions using both simple and chance devices and Tinkerplots. By examining the results produced using these resources, the unit develops students' ideas around probability and the structure of the devices producing these trends, also called theoretical probability. Students will discover how experimental and theoretical probability produce similar values.

Essential Questions

- How is theoretical probability related to experimental probability?
- How are probability and statistics used to solve real-world problems?

Mathematical Concepts

- Probability = Number of desired outcomes/number of possible outcomes
- Probability is a mathematical way of calculating how likely an event is to occur. The probability of an event occurring is defined as the ratio of desired outcomes to the number of equally likely total outcomes.
- Theoretical probability is probability-based on an ideal situation. The theoretical probability of an event remains the same no matter how events turn out in the real world. It is based on the structure of the process and its possible outcomes.
- Experimental, or empirical probability is based on observed data generated by the event, experiment, or chance process. To get the best estimate of probability, the experiment should be repeated many times.

Unit Overview

Because events can't be predicted with total clarity, the best we can say is how likely they are to happen. Probability is a numerical description of how likely an event is to occur. Building mathematical models to simulate events help students see the connections that help experts make statistical inferences.

Teacher Guide

Exploring Expectation

Day 1: Activities: Melissa's Spinners and Sneaky Pete

Day 2 Activities: What is Theoretical Probability and Mystery Spinners

From Rich Leyers's Data Modeling Unit

What Does Probability Mean Anyway

Materials and Preparation

Transparent Spinners (or two-color and four-color spinners)
Online spinner (https://www.nctm.org/adjustablespinner/)
Computers with TinkerPlots
Week 1 Slide deck
Worksheets –

1) *What Does Theoretical Probability Really Mean Anyway?*
2) *TinkerPlot Instructions*

Instruction

Day 3: Activities

Students have been examining probability to measure chance and find the theoretical probability using simple spinners. Students should have also noticed that theoretical probability does not always match what we actually observe when collecting a sample (experimental probability). Today students compare theoretical probability and experimental probability (also called empirical probability) using a four-color spinner.

Whole Group

1. We have been talking about probability as a way to measure chance and have been finding the theoretical probability of simple spinners. We have also noticed that our theoretical probability does not always match what we actually observe when we collect a sample (experimental probability). Today we will see a situation where theoretical probability tells us we should expect to observe a sample that isn't even possible!!!
2. So, how is theoretical probability related to experimental probability?

Distribute handout *What Does Theoretical Probability Really Mean Anyway?*

and transparent or two-color spinners.

3. If you spin this two-color spinner, how many times does theoretical probability tell us we should expect to land on yellow?

Allow students to spin and record data in the chart on their worksheet.

4. What were your results? How did they compare to what you predicted?

Distribute four-color spinners.

5. Now let's look at a probability scenario that is a little different. Turn the worksheet over. How is this spinner different?
6. Does this change our theoretical probability of landing on yellow?
7. Now predict again. Out of 10 spins, how many yellows are possible? How many are likely? How many are not likely?

Allow students to spin and record data in the chart on their worksheet.

Day 4 Activities: Investigating Sampling
Students continue to use data collected using spinners on Day 3 to compare experimental probability to theoretical probability. A class graph of outcomes is constructed. The graph is used to frame conversations related to predictions, likely outcomes, unlikely outcomes, and their relationship to the theoretical probability

Whole Group

1. Get out your data from yesterday. Look at the sample of 10 spins you collected yesterday using the two-color spinner. What is the experimental probability based on your sample?
2. What do you expect other students' samples to look like?
3. Imagine we combined all of our results for the number of yellow. Draw a visual display of what you think the numbers of yellow in 30 samples would look like. For example, how many samples would have 5 out of 10? How about 2 out of 10?
4. How could we display everyone's data in one place?
5. Is there a way that would help us see trends or patterns?

Allow students to construct a class histogram using sticky notes for each student's data.

Think-Write-Pair-Share:

a. Did this graph look like your prediction?
b. What do you notice about these outcomes?
c. Which outcomes are likely?

   d. Which outcomes are unlikely?
   e. How are these outcomes related to the theoretical probability?
6. The theoretical probability of spinning yellow is ½. This means in a sample of 10, we should expect to observe 5 yellow.
7. But...we can see that lots of samples didn't have 5 yellows! So what does this mean? How many didn't have any yellow?
8. Look at our data again. What else do you notice?

Students may notice the shape of the data, range, or outliers. Documenting student thinking on the board will help this conversation.

9. Now let's look at the data we collected using the four-color spinner. What is the experimental probability based on your sample?
10. How does your experimental probability compare with the theoretical probability?
11. What do you expect other students' samples to look like? What do you think a visual display of all of our data would look like?
12. How could we display everyone's data in one place?
13. Is there a way that would help us see trends or patterns? (Hint Hint - we just did this with the 2 color spinner)

Allow students to construct a second histogram using sticky notes for each student's four-color spinner data.

14. We calculated theoretical probability. The theoretical probability of spinning yellow on this spinner is ¼. This means in a sample of 10, we should expect to observe 2.5 yellow.
15. But...2.5 yellow is not even possible! So what does this mean?

Students may suggest rounding up to 3 or down to 2.

16. Did you notice that although there's variability in the outcomes, they are all centered around 2.5?
17. Based on what we saw today, explain the difference between theoretical probability and experimental probability.

Day 5

Activities: Introduction to TinkerPlots
Students have been using manual spinners to collect samples, but this would not be effective for modeling samples for large data sets. The introduction of TinkerPlots today to sample results of 100 spins of a two-color will prepare students for using Tinkerplots for birthrate data next week.

Think-Pair-Share - Share the following scenarios with students

Sarai said "theoretical probability doesn't tell me anything, because out of 10 spins I can get anything! Not just 5."

Ask: Do you agree? What would you tell Sarai?

Instructions: Partner A plays the role the Sarai - Partner B, agree or disagree with Sarai and justify your reasoning. Partner A, please feel free to disagree or ask for further explanation from Partner B. Be nice though, it will be your turn next. Give students 30-60 seconds to complete before announcing the next scenario. Partner B will play Cynthia.

Cynthia says the probability of her spinner landing on A is ¼. If she spins the spinner 18 times she expects it to land on A 4 ½ times.

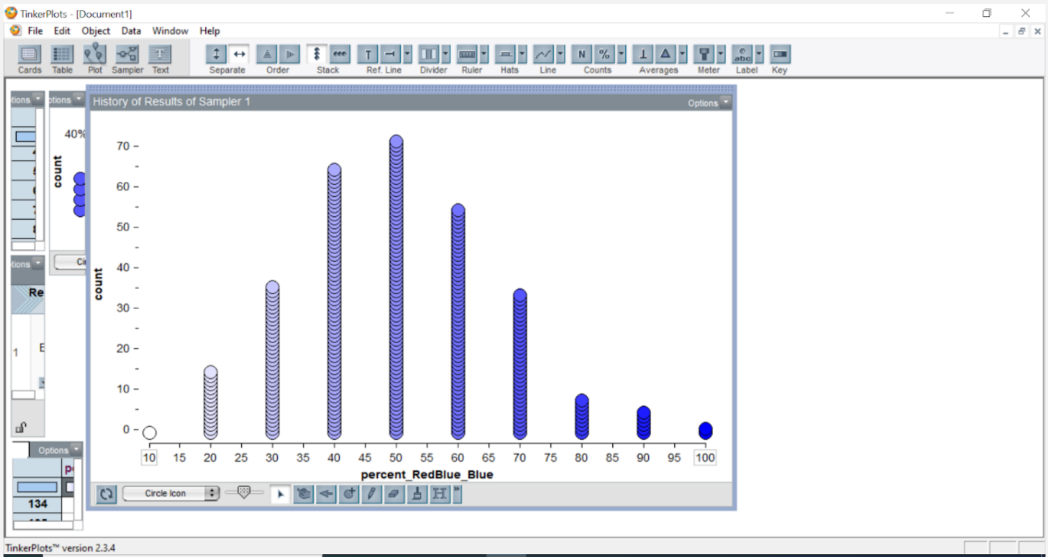
Ask: What does Cynthia mean by 4 ½ ? How did she get 4 ½?

Whole Group

1. We have been spinning a lot of spinners this week! But what if we want to collect a VERY LARGE sample? We could do 1000 physical spins for homework …
   OR we could get our computers to help. Let's learn how to do this with TinkerPlots!

Have students log on to TinkerPlots to build spinners and practice collecting samples.
Distribute worksheet: *TinkerPlot Instructions*

2. Do you think the TinkerPlots spinner and the hand spinners give the same results? Would you be okay combining samples from the TinkerPlots spinner to samples with a handheld spinner like we combined our samples across the class?
3. What do you think the TinkerPlots spinner might be helpful with as we explore probability?



4. Did you notice that although there's variability in the outcomes, they are all centered around 5? Wait! What is variability?

Students may recall measures of center. Prompt discussion, "Why didn't everyone get five yellows? What do we mean by variability?"

Introduction to Birthrates

Materials and Preparation

Transparent Spinners (or two-color and four-color spinners)
Online spinner (https://www.nctm.org/adjustablespinner/)
Computers with TinkerPlots
Week 2 Slide deck
Worksheets –

1) *Thoughts about Finding Something Strange with Gender*
2) *North Carolina Data*
3) *Use Your Procedure*
4) *Use Another Procedure ??? Other Groups Procedures*

Day 6

Activities: Introduction Birth Rates & What is a model?

Discuss examples of models. Think about the purpose of physical models and consider how mathematical models serve a similar purpose. A probability model represents the likelihood of different outcomes of something, and can be used to simulate samples.

Students use ideas about probability to help make decisions about the real world.

Sex in humans is determined by a random process where DNA from a mother and DNA from a father create chromosome pairs that determine if a child is male or female. We are going to see how probability can help us analyze birth data. Look at state data (Local data may be available at https://wonder.cdc.gov/natality.html).

Whole Group

1. Why does this idea of chance matter?
2. Think of some real-world examples (besides spinners and dice)

In your journal - Think of as many examples in as many different fields of study as you can! Write or draw them in your journal.

3. What are some examples of models you listed? *Record the student ideas.*
4. What makes these a good model?

Possible responses

- Is a representation that helps us study or understand the real thing

- Chooses some aspect of that "thing" to highlight
- May ignore some aspects of that "thing"
- Corresponds to the "thing" in some meaningful way

5. Let's think about the two-color probability scenario that we have discussed before.
6. What real life scenario could this be a model of?
7. What kind of scenario could you use it to simulate?

Explain - A probability model represents the likelihood of different outcomes of something, and can be used to simulate samples.

- It is a representation that helps us study or understand the real thing
- Chooses some aspect of that "thing" to highlight
- May ignore some aspects of that "thing"
- Corresponds to the "thing" in some meaningful way

As we have seen, our theoretical probability and experimental probability are often not the same. Each time we draw a sample, our experimental probability is likely to change. It's random. This is called "sample-to-sample variability." As the random sampling is repeated, the statistic will vary.
Although there are many possible outcomes when you take a sample, we have also seen that some outcomes are more likely than others. Today we will build probability models to explore how likely different outcomes are in different situations. We are going to see if we can use our ideas about probability to make decisions about the real world! Every day babies are born all over the world. When babies are born, nurses record different variables for official records, and researchers and doctors use these records to look for things that might be out of the ordinary.

8. If you looked at birth statistics for a county, what percentage of males and females would you expect to be born in a one-year sample?
9. If a county has 10 babies born each year (a small county!), how many do you expect to be female? Why?
10. Do you think you would observe this expectation in every sample of 10?

Show students birth rate data for boys and girls by county in your state.

11. Do any of these proportions lead you to wonder if something other than random chance was determining the sex of the children?

Arrange students into small groups (3-4 students)

12. What ideas do you think you would need to make a procedure that can help identify abnormal proportions of males or females in a sample?

13. How do you think probability might be able to help you in this project?
14. If you had to create a procedure for county officials to use to identify abnormal proportions of males or females in a sample, what would it be? Talk with your group, but you don't have to record the exact same thing as your group members.

Day 7

Activities: Introduction to Birthrates: Modeling Birthrate Data

What ideas do you think you would need to make a procedure that can help identify abnormal proportions of males or females in a sample?
Share Tennessee birthrate data. Allow students time to identify any counties where something might be going on with children's sex at birth that is out of the ordinary.
Use TinkerPlots to create a probability model to simulate births.

Whole Group
Remember your job this week. You are working for the state health department, and your job is to develop a procedure that doctors and nurses can follow to determine if normal percentages of males and females are being born in their county, or if the percentages suggest something strange might be happening. Today we are going to explore how a probability model might help you create this procedure. Let's look at real data collected for births in Tennessee.

Distribute *Thoughts about Finding Something Strange with Gender*
Allow students to look at data independently before discussing in their groups.

Silently, look at the data for counties in Tennessee. Identify any TN counties where you think something might be going on with children's sex at birth that is out of the ordinary. Make notes about why you think something strange might be happening.

1. What counties did you think may have something strange happening? Explain.
2. What ideas do you think you would need to make a judgement about how normal these gender proportions are?

Wouldn't it be nice to observe 200 years of samples to see what kinds of proportions are likely, just from chance sample-to-sample variation. But none of us are going to live that long! Instead, let's create a probability model that we can use to SIMULATE 200 years of births! Hmmm…What tools have we used over the past few weeks that might help us do this? TINKERPLOTS!

In your group, create a model that can simulate the sex of individual babies across repeated trials. We want to simulate repeated yearly samples from a small county that only has 10 births a year. Set up your model, simulate 200 years, and graph the percentages of baby boys for each of the years across 200 different years (with 10 births per year).

3. What do you notice about the simulated outcomes from your model?
4. What was the average of the simulated 200 years? Why?
5. Where did most of the simulated 200 years fall? Why?
6. Do all of our simulated 200 years look alike? What does this tell us?
7. Does your simulation look like your neighbor's?
8. In your journal, Predict: What will our data look like if we simulate 200 years with 100 births per year? Think, draw a simple sketch, discuss your ideas with your group.

Day 8

Activities: Comparing Model Data to Real Data, Inventing Procedures

Students use TinkerPlots to create a model that can simulate the sex of individual babies across repeated trials. How could student's models help us think about a procedure to identify problematic proportions of males and females in a sample for small, medium, and large counties.

Whole Group

1. What are some of your ideas about the data for a county with 100 births for 200 years?

Let's create a model that can simulate the sex of individual babies across repeated trials. We want to simulate repeated yearly samples from a medium county that only has 100 births a year. Set up your model, simulate 200 years, and graph the percentages of baby boys for each of the years across 200 different years (with 100 births per year).

2. What do you notice about the simulated outcomes from your model?
3. How do they compare to the small county?
4. What is the average for the simulated 200 years in the medium county?
5. How does that compare to the small county??
6. Where do most of the simulated 200 years fall in the medium county?
7. How does that compare to the small county??

We have looked at small counties with 10 births a year and medium counties with 100 births a year. Look back at your predictions for the medium county. Think about the differences we saw between the small county and the medium county. What will our data look like if we simulate 200 years with 1000 births per year? Draw a quick sketch of your prediction in your journal.

Create a model that can simulate the sex of individual babies across repeated trials. We want to simulate repeated yearly samples from a large county that only has 1000 births a year. Set up your model, simulate 200 years, and graph the percentages of baby boys for each of the years across 200 different years (with 1000 births per year).

8. What do you notice about the simulated outcomes from your model?
9. How do they compare to the small and medium counties?
10. What does one dot represent on each line plot?
11. What is the average for the simulated 200 years in the large county?
12. How does that compare to the small and medium counties?
13. Where do most of the simulated 200 years fall in the large county?
14. How does that compare to the small and medium counties?
15. As the sample size gets bigger, what happens to the sample-to-sample variability?
16. If you observed one year of real data with 10 births, and it had 70% female, would that worry you? Why or why not?
17. If you observed one year of real data with 100 births, and it had 70% female, would that worry you? Explain your answer.
18. If you observed one year of real data with 1000 births, and it had 70% female, would that worry you? How did you reach your conclusion?

Think about our job for the week. Your team is supposed to create a model to determine when health professionals should be concerned about birth rates of boys and girls.

19. How do the models we constructed today help us think about a procedure to identify problematic proportions of males and females in a sample?
20. What information would a procedure need to use?

Think about the TN data. Based on the birth rate data…How would you describe a small county? A medium county? A large county?

In your groups look at the birth rates from different counties in North Carolina in 2004. Explore this data with your group to determine if the percentage of male babies in any of the counties suggests that something is happening in that county to make males more likely than females. You can use the data in this table, the models you built earlier, or any new models you think would be helpful to support your claims.

21. Does this data support the claim that in some counties something strange might be happening to make boys more likely than girls?

22. Name the counties below that you think show a normal proportion of boys, and the counties where something strange might be happening.
23. Explain your group's thinking for each of the categories.
24. Does the overall state percentage seem normal? Why?

Okay, we have spent a lot of time looking at patterns in the data and your job is to invent a procedure to help identify unexpected proportions in the birth rates of boys and girls. The State Department of Health is anxious to know if there is a problem with the birthrate and your team presentation is essential. With your group, create a procedure that others could follow that would help them identify proportions of males and females that are possibly problematic because they might not just be due to chance.

Doctors and nurses across many counties need to be able to use this, so make sure and be as clear and detailed as possible about your procedure.

Think - What are you trying to do? What information do you need?

Plan - Highlight the steps to your procedure

Communicate- Tell your group

Day 9 & 10

Activities: Sharing Procedures, Using Procedures, Exploring more Real Data, Revising our Models
Distribute *Use Your Procedure*
Allow students time to complete their invented procedures. Instruct students to use the worksheet to try their procedures using data from TN, AR or NC.
Student groups share/present procedures to the class.
Distribute *Use Another Procedure*
After each presentation, allow students to

a. Ask clarifying questions,
b. Use the handout to document the procedure.
c. Try the procedure.
d. Record the results after trying the procedure.

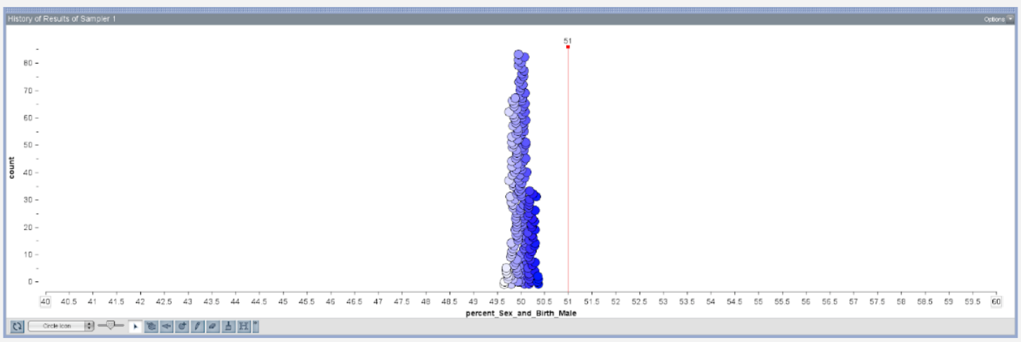
Whole group

1. What things must you consider when you are looking for "something strange"?
   Write student ideas to extend the conversation
      Sample Size
      Deviation from the Expected

2. What is happening here?

State	% Female	% Male
Arkansas	49%	51%
Tennessee	49%	51%
North Carolina	49%	51%

3. How likely is this to happen by chance?
4. IF we built a Tinkerplots model with 100,000 births per year and simulated 200 years…
5. What do you think would the average be % male?
6. Where do you think would most of the % male observations fall?
7. What do you think would the biggest and smallest % male be?



8. What does our model tell us about 51% males in NC, TN, or AR?

Our models assumed that the probability of observing a female birth is ½

9. After looking at the data and hearing this story, should we make any changes to our model?
10. Is the relationship between theoretical and experimental a 2-way relationship?

Introducing Compound Probability

Materials and Preparation

Transparent Spinners (or three-color spinners)
Online spinner (https://www.nctm.org/adjustablespinner/)
Computers with TinkerPlots
Week 3 Slide deck
Worksheets –

1) *Investigating Compound Probability*
2) *Predicting the Results of 180 times*
3) *Game on the Line*
4) *Toms' Double Spinners*
5) *Get to Work on Time Datasheet*

Day 11

Activities: What is the theoretical probability of different outcomes based on sample space?

Whole Group
Rolling a dice and flipping a coin at the same time is a compound event. Flipping a coin is independent of rolling a dice. Based on the statements above, respond to this question in your journal. After giving students time to record their thoughts.

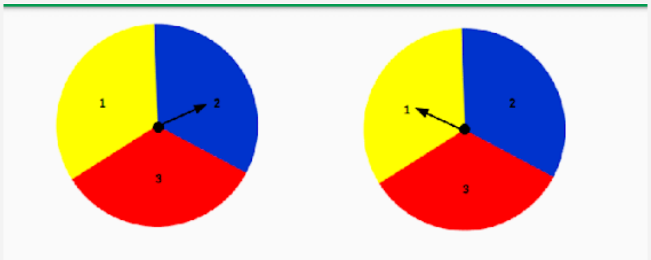
1. What do you think the words "independent" and "compound" mean in mathematics?

Compound means more than one. A compound event means there is more than 1 one thing happening at the same time.
Independent means that the outcome of one thing happening does not affect the outcome of another thing happening.

2. How does this fit with what we know about probability?

Compound probability is the likelihood of two independent events occurring. Let's use our spinners to examine compound probability.

Distribute 2 three-color spinners to each student group and the *Investigating Compound Probability* worksheet.



This time we will spin two spinners at the same time and then the two numbers the arrows are pointing at will be added together.

3. If we spin the spinners at the same time and add the values that appear on each spinner, what are all of the possible outcomes (sums)? Write down all the possible sums on your handout!

Think about what would have to happen with each spinner to get each sum.

4. Which sums are not possible?
5. Are all sums equally likely?
6. What are the possible pairs and outcomes for the sum of the two spinners?

Allow students time to work on this using the handout and spinners.

7. What do you think the theoretical probability of getting a sum of 4 is? Hmmm… How can we use what we learned about theoretical probability last week to figure this out?

Look at your chart of possible outcomes on the handout.

8. Which of the sums in the table are our target outcomes?
9. Is a sum of 4 from 1,3 the same as a sum of 4 from 3,1? What about 2,2 vs. 2,2?
10. How does this impact our theoretical probability?

Day 12

Activities: Sample space, Making predictions

Whole Group

In probability theory, the sample space of an experiment or random trial is the set of all possible outcomes or results of that experiment.

There is a glass jar that contains 8 red marbles, 6 green marbles, 2 blue marbles and 4 yellow marbles. If you were to pull one marble out of the jar...

Investigating Chance

1. In your journal write down: How would we describe the sample space in this situation?

Sample space is the set of all possible outcomes {red, green, blue, yellow}

2. What is the probability of each outcome?
3. What is the sum of the probabilities? Is that always true?

What if we have 2 jars of marbles? We have a compound event. How many possible outcomes are there now?

4. What would a sample space look like if we were to randomly select one marble from each of our jars at the same time?

Think about how you came up with this answer. Write it or draw it in your journal!

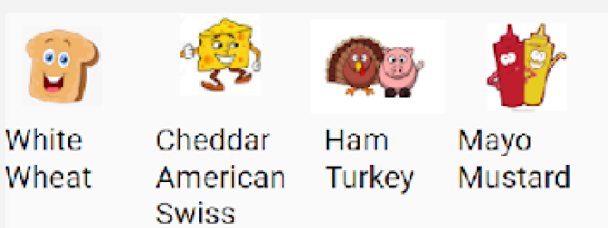
5. What is the probability of randomly choosing a green marble from both jars?
6. What is the probability of randomly choosing a red marble from one jar and a yellow marble from the other?

Emphasize counting out the sample space, counting the total number of outcomes, and total number of target outcomes. Connect to the table or tree diagrams if is seems helpful to the students.

Challenge questions:

7. What is the probability one of them is yellow?
8. What is the probability of NOT choosing a yellow from either jar?
9. What is the probability of not getting a pair of green marbles?

Remember finding the possible pairs of results for the spinners yesterday? The lists described our sample space. But What if we weren't dealing with numbers (order comes naturally) and such a small sample space like our marble jars? Is there a way to show all the possible outcomes clearly? Let's make sandwiches!



10. How many different sandwich combinations are there if you can choose one bread, one cheese, one meat, and one sauce (must include all 4) from the following options?

Quickly, create a visual showing how you figured out the sample space (possible pairs of results) from the spinner scenario. Allow students to show how they determined the sample space. Discuss the advantages of different ways. Table, list, tree diagram, etc.

11. What is the relationship between sample space and theoretical probability?
12. Now think back to our spinners. Imagine we were only interested in figuring out the theoretical probability of getting a sum of 4 when spinning our 2 spinners 180 times.
13. How would we figure out the theoretical probability of getting a sum of 4?

If you think of the number of 4's you expect and divide that by the total number of 2's, 3's, 4's, 5's, and 6's (180), we call that a probability.

14. What is your guess about the probability of a sum of 4?
15. How can we use our predictions to figure out the theoretical probability of one outcome?
16. Does it matter that some outcomes have more than one possible way of occurring?

I found these two ways of showing how to calculate spinner sample space.

17. Which table do you think shows the correct sample space?
18. Which shows all the possible pairs of results?

A

Spinner 1	Spinner 2	Outcome (Sum)
1	1	2
1	2	3
1	3	4
2	1	3
2	2	4
2	3	5
3	1	4
3	2	5
3	3	6

B

Combination (addends)		Outcome (sum)
1	1	2
1	2	3
1	3	4
2	2	4
2	3	5
3	3	6

Here, play the role of thinking B is correct and position students to use TP to convince you A is correct. Have them write out the expected observations out of 180 samples for BOTH...then design a plan to use TP to test which is more consistent with the TP outcomes. *This is also a good time to have more conversations about*

Let's use Tinkerplots to run the spinners 180 times. Record your results in the table. You need to edit the table on the handout since we can't seem to agree :).

19. Thinking about the probability of getting a sum of 4 again, what do your results suggest the probability should be?
20. How can you use these Tinkerplots outcomes to convince me that 1,2 and 2,1 are different in the sample space?

Think about how could we graph all our outcomes for sum of 3? What do you think that would look like? What if we increase the number of trials? Do you have enough space to record 180 trials?

21. What do you predict would happen if we did another 180 spin trial in TinkerPlots?
22. What if we did a 360 spin trial? 540 spins??

I want you to think about something and write your ideas in your journal. Does the difference between theoretical and expected frequencies (as a proportion of the total) tend to decrease as the number of spins increases? What do you notice about the difference between your predicted and observed outcomes as the number of spins increases? How does this fit with what we know about compound probability, sample spaces, sample sizes, and the relationship between them?

Remember how I told you my smaller chart was better? I'm still not convinced I was wrong, because I saw close to 60 out of 180 for sum of 4 on most of yours. And on a few screens I saw around 30 3's. You all said it should be 40 3's. I think we should make our own display based on how many 3's you got out of 180.

Day 13

Activities: Game on the Line, Assessment

Game on the line from Rich Leyers's Data Modeling Unit

Whole Group: Game on the line extension
In TinkerPlots, build a model of Jena shooting three free throws. Use your model to create simulated data that can help you estimate the probability that she misses all three shots. Make a graph in TinkerPlots of the simulated data you collected with your model

Use tree model to help students describe the sample space.

Day 14

Activities: Tom's Double Spinner, Make it on Time!

Tom's Double Spinner from Rich Leyers's Data Modeling Unit

Whole Group: A random teacher, not me, drives through 4 traffic lights on her way to work. The 4 lights are not programmed with each other, which means the number of red lights that this teacher, that really is not me, hits is always random chance. The number of red lights she hits makes a big difference in the time it takes me, I mean HER, to get to school! This teacher told me that she likes to lay in bed and think about her day before she actually gets up and ready for the day. Her morning daydreaming sometimes causes her to be late. I told her you all were probability experts and I even confessed that you proved me wrong on the compound probability chart! Maybe you can help me determine how early one would have to leave to get to work on time…you know, so I can tell her.

Investigating Chance

Here is what we know

Distribute *Get to Work on Time Datasheet*

Implementation