Practice Test Solutions

1. $C_2H_4(g) + H_2O(\ell) \rightarrow C_2H_5OH(\ell)$

$+68\,kJ/mol - 237\,kJ/m \rightarrow -175\,kJ/mol$

$G_{Products} - G_{reactant} = (-175 - 68 + 237)\,kJ$

$= -6\,kJ.$

$G < 0 \Rightarrow$ spontaneous.

[expect a bit more in a real test problem!]

2. (a) $m_N c_N \Delta T_N + m_w c_w \Delta T_w = 0$ (assume room $T = 25^\circ C$)

$(0.34\,kg)\left(1800\frac{J}{kg}\right)(T_f - 25) + 1500\,kg\; 4186\frac{J}{kg}(T_f - 100^\circ C) = 0$

$612\,T_f - 15300 + 6279\,T_f - 627900 = 0$

$T_f = 93.3^\circ C \;[366.3K]$

(b) Noodles $dS = \int_{T_{i_N}}^{T_f} \frac{dQ}{T} = \int_{T_i=298}^{T_f=366.3K} \frac{mc\,dT}{T} = 0.34 \times 1800 \ln\frac{366.3}{298}$

$= 2585\frac{J}{K}$

Water $dS = \int_{T_i=373}^{366.3} \frac{mc_w\,dT}{T} = m_w c_w \ln\frac{366.3}{373} = -113.8\frac{J}{K}$

$\Delta S_{TOTAL} = \Delta S_w + \Delta S_N = \underline{2471\,J/K}$

3. (a) $V = \frac{NkT}{P} = \frac{nRT}{P} = \frac{3 \times 8.314 \frac{J}{mol\,K} \times 300K}{1.01 \times 10^5 \frac{N}{m^2}} = 0.074\ m^3$

(b) Rigid box $\Rightarrow C_v = \frac{3}{2}R$ /mol.

$$\Delta Q = nC_v \Delta T$$
$$= 3 \times \frac{3}{2}R \times (400 - 298)$$
$$= 3816\ J$$

(c) $P = \frac{nRT}{V} = \frac{3 \times 8.314 \frac{J}{K} \times 400\,K}{0.074\,m^3} = 1.35 \times 10^5\ Pa$

(d) $\Delta S = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc \ln \frac{T_f}{T_i}$
$$= (3 \times \frac{3}{2}R) \ln\left(\frac{400}{298}\right)$$
$$= 11\ J/K.$$

(e) $$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$U = \frac{3}{2}nRT = \frac{3}{2} \times 3 \times 8.314 \times 298 = 11.1\ kJ$$

$$\left(\frac{4\pi mU}{3Nh^2}\right)^{3/2} = \frac{4 \times 3.14 \times 4 \times 1.67 \times 10^{-27} \times 11.1 \times 10^3}{3 \times 6.022 \times 10^{23} \times (6.626 \times 10^{-34})^2}$$
$$= 4 \times 10^{31}$$

$PV = NkT$

$\frac{V}{N} = \frac{kT}{P}$

$$\frac{V}{N} = \frac{kT}{P} = \frac{1.381 \times 10^{-23} \frac{J}{K} \times 298\,K}{1.01 \times 10^5\ Pa} = 4.07 \times 10^{-26}\ m^3$$

$$\ln(\quad) = 14.3$$

$$S = Nk\left[14.3 + \frac{5}{2}\right]$$

$$= 3 \times N_A \times k\,[16.8]$$

$$= \underline{419\ \text{J/K}}$$

4. (a) $e = 1 - \frac{T_C}{T_H} = 1 - \frac{250}{300} = 0.166$

ie 16.6% efficient.

(b) $e = \frac{W}{Q_H} \Rightarrow Q_H = \frac{W}{e} = \frac{1\text{kJ}}{0.166} = \underline{6000\text{J}}$

(c) $COP = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{1}{\frac{Q_H}{Q_C} - 1} = \frac{1}{\frac{T_H}{T_C} - 1}$

$$= \frac{T_C}{T_H - T_C} = \frac{250}{300 - 250} = \underline{5}$$

(d) High COP is better. $Q_C = COP \times W \Rightarrow$ high COP means we get heat extraction being COP multiplied by W.

5. (a) $Z = \sum_s e^{-\beta E} = e^{-\beta\mu B} + e^{+\beta\mu B}$

(b) $\mu B = 0.4\frac{\text{eV}}{\text{T}} \times 0.1\,\text{T} = 0.04\,\text{eV}$

$$P_\downarrow = \frac{e^{-\mu B \beta}}{Z} = \frac{e^{-0.04\times 29}}{e^{-0.04\times 29} + e^{+0.04\times 29}} \qquad \beta = \frac{1}{RT} = \frac{1}{k400} = 29\ eV^{-1}$$

$$= \frac{0.313}{0.313 + 3.192}$$

$$= \underline{0.089}$$

$$P_\uparrow = \frac{e^{+\mu B \beta}}{Z} = \frac{3.192}{0.313 + 3.192}$$

$$= \underline{0.911}$$

(c) $$\bar{E} = \epsilon_\downarrow P_\downarrow + \epsilon_\uparrow P_\uparrow$$

$$= 0.04\times 0.089 + -0.04\times 0.911$$

$$= \underline{-0.0329\ eV}$$

6. (a) Nb → volume of squished together atoms — Repel if they touch.

$\frac{aN^2}{V^2}$ → attraction between atoms

(b) $b \approx r^3$ → radius of a neon is $(2.8\times 10^{-29})^{1/3} = \underline{3\times 10^{-10}\ m}$

[Actual exam question would have more on vd Waals eqn]

7. $F = U - TS$

Take differential

$$dF = dU - TdS - SdT$$

Now use $dU = TdS - PdV + \mu dN$

$$dF = (TdS - PdV + \mu dN) - TdS - SdT$$

$$dF = -PdV - SdT + \mu dN$$

$\Rightarrow$ T, N constant $\left(\frac{\partial F}{\partial V}\right)_{T,N} = -P$

V, N " $\left(\frac{\partial F}{\partial T}\right)_{V,N} = -S$

V, T " $\left(\frac{\partial F}{\partial N}\right)_{V,T} = +\mu$

(b) $$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = +\frac{\partial}{\partial T} kT \ln\left(2 - \frac{\epsilon}{kT}\right)$$

$$= k \ln\left(2 - \frac{\epsilon}{kT}\right) + \frac{2T - \frac{\epsilon}{k}}{}$$