Sample Test Solution Test 2

1. (a) A microstate is one specific configuration of the system whereas a macrostate is a collection of states that share a given property. In to 100 coins example a microstate would be coin #1 = heads and all the rest tails. Another microstate would by coin #2 = heads and all the rest tails. Both of these microstates are part of the same macrostate along with all the other microstates (98 more of them) with one coin a head and the rest tails.

The multiplicity is the number of microstates in a given macrostate.

(b) The multiplicity of all heads up is 1.

(c) 1T 99H. Multiplicity is 100 (see part (a))

(d) Largest multiplicity when 50H and 50T.

$$\Omega = \frac{100!}{50!\,50!} = \frac{2.82\times10^{92}}{3.04\times10^{64}} = \underline{\underline{0.92\times10^{28}}}$$

(e) $S = k\ln\Omega$

$\Omega = 1 \Rightarrow S = \underline{0}$

$\Omega = 100 \Rightarrow S = k\ln 100 = \underline{4.61k}$

$\Omega = 0.92\times10^{28}$ $S = \underline{64.4k}$

2.

q_A	Ω_A	q_B	Ω_B	Ω_{TOTAL}
0	1	4	5	5
1	2	3	4	8
2	3	2	3	9
3	4	1	2	8
4	5	0	1	5
				35

— $q_A = 3$ $q_B = 3$ is the most likely final state.

— $q_A = 4$ is only found in $\frac{5}{35} = \frac{1}{7} \Rightarrow 14.2\%$

3. (a) Derived by considering all of the possible states available to an atom in a volume V. The energy U sets a limit on the possible momenta that the particles can take up. Quantum mechanics steps in to determine the "size" of a state via $\Delta x \Delta p_x \sim h$. Counting the states gives the multiplicity Ω. $S = k \ln \Omega$ which gives the Sackur-Tetrode equation.

(b) $n = 1$ He has atomic mass $4 \times 1.67 \times 10^{-27}$ kg $= 6.68 \times 10^{-27}$ kg

$$U = \frac{3}{2} N_A k T = \frac{3}{2} nRT = \frac{3}{2} \times 1 \times 8.314 \times 500\text{K} = \underline{6236\text{J}}$$

$$\frac{V}{N_A} = \frac{kT}{P} = \frac{1.381 \times 10^{-23}\ \text{J/K} \times 500\text{K}}{1.01 \times 10^5\ \text{Pa}} = 6.84 \times 10^{-26}\ \text{m}^3$$

$$N_A k = R$$

$$S = R\left[\ln\left(6.84\times10^{-26}\left(\frac{4\pi\, 6.67\times10^{-27}\times 6236}{3\times 6.02\times10^{23}(6.6\times10^{-34})^2}\right)\right) + \frac{5}{2}\right]$$

$$= R\left[\ln(1.17\times10^{6}) + \frac{5}{2}\right]$$

$$= \underline{\underline{137\ \frac{J}{K}}}$$

(c) Double N but also double U because the new He atoms are at the same temperature

$$S_{2N} = 2Nk\left[\ln\left(\frac{V}{2N}\left(\frac{4\pi m(2U)}{3(2N)h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$= 2Nk\left[\ln 2 + \left[\ln\left(\frac{V}{N}\left(\text{FRED}\right)^{3/2}\right) + \frac{5}{2}\right]\right]$$

$$S_{2N} - S_N = 2Nk\ln 2$$

4. $m = 1\,kg \quad c = 902\,J/kg \quad L_{Al} = 321\,kJ/kg.$

$$S_{heat} = \frac{dQ}{T} = \int_{T=293K}^{T=933} \frac{mc\,dT}{T} = mc\left[\ln T\right]_{293}^{933}$$

$$= 1\times 902\ln\left(\frac{933}{293}\right) = \underline{\underline{1045\ J/K}}$$

$$S_{melt} = \int\frac{dQ}{T} = \frac{1}{T}(mL) = \frac{1\times 321\times10^{3}}{933}$$

$$= 344\ J/K.$$

$$\Omega = e^{S/k}$$

$$\Omega_{heat} = e^{\frac{1045}{1.381\times10^{-23}}} = e^{7.6\times10^{25}} = 10^{3.3\times10^{25}}$$

$$\Omega_{melt} = e^{\frac{344}{k}} = e^{2.5\times10^{25}} = \underline{\underline{10^{1.08\times10^{25}}}}$$

5. (a) From the thermodynamic identity with N and V fixed

$$dU = TdS - P\cancel{dV}^{0} + \mu \cancel{dN}^{0}$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N}$$

$$\Rightarrow \quad \frac{1}{T} = \frac{\partial\left(2Nk\ln\left[VU^{0.5}\right]\right)}{\partial U}$$

$$= 2Nk\left[0.5\ln U + \ln V\right]$$

$$\frac{1}{T} = \frac{Nk}{U}$$

$$\boxed{U = NkT}$$

(b) Again from the thermodynamic identity with U, N fixed

$$\cancel{dU}^{0} = TdS - PdV + \mu \cancel{dN}^{0}$$

$$P = T\left(\frac{\partial S}{\partial V}\right)_{U,N}$$

$$= T\frac{\partial}{\partial V}\left[2Nk\left[0.5\ln U + \ln V\right]\right]$$

$$= T\frac{2Nk}{V}$$

$$\boxed{P = \frac{2NkT}{V}}$$